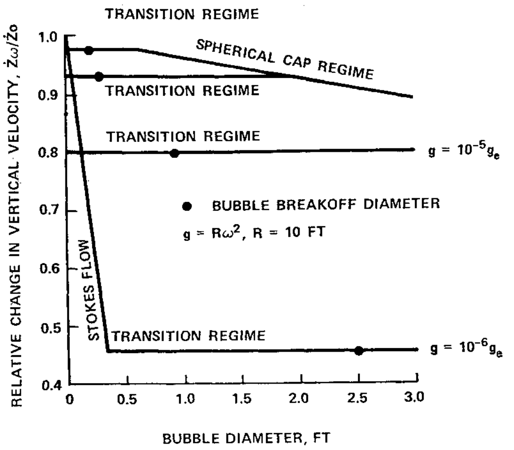

Fig. 10 Change in velocity of bubbles in liquid hydrogen towards the center of rotation in a rotating laboratory.

The rotational speed is expressed as $\omega = (a/R)^{1/2}$ where R is the distance measured from the center of rotation to the vicinity of the bubble. It is assumed here that R is essentially constant over the distance traveled by the bubble.

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Effect of Nozzle Throat Radius of Curvature on Gasdynamic Laser Gain

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RAPID supersonic, nonequilibrium expansions of hot, vibrationally excited $CO_2-N_2-H_2O$ or He mixtures can produce population inversions and hence laser action in CO_2. This is the essence of the CO_2-N_2 gasdynamic laser. Interest in gasdynamic lasers is rapidly growing, and discussions in this regard can be found in Refs. 1–4, among others.

The heart of a typical gasdynamic laser is a bank of extremely short, minimum length contoured nozzles in which the N_2 vibrational energy is essentially frozen in the throat region. Such a sharp-cornered nozzle is sketched in Fig. 1a). Experimental and theoretical studies of the aerodynamic flow through such nozzles are described in Refs. 5 and 6. In particular, Greenberg et al.[5] clearly point out that the subsonic radius of curvature R_c (upstream of the sharp corner throat) should be made small, $R_c \approx h^*$, in order to enhance vibrational freezing and the subsequent laser gain.

This Note complements the study of Ref. 5 by examining the effect of extending the radius of curvature into the supersonic section, as sketched in Fig. 1b). By keeping the throat height constant and completely rounding the sharp corner, the nozzle of Fig. 1b) is easier to manufacture and align than the nozzle of Fig. 1a). However, this results in less vibrational freezing and hence reduced laser performance. The purpose of this Note is to indicate quantitatively the compromise in laser performance due to such rounding of the throat.

The present results were obtained from numerical experiments employing the time-dependent nonequilibrium nozzle flow analysis of Refs. 7 and 8. This analysis fully couples the vibrational kinetic rate equations with the governing quasi-one-dimensional flow equations of continuity, momentum and energy. Nonequilibrium conditions are included both upstream and downstream of the nozzle throat. A simplified vibrational model is used which approximates the detailed molecular energy transfer. The results of Refs. 9 and 10 show that the model and analysis of Ref. 7 yield reasonable agreement with experimental data for gasdynamic laser gain.

For the present investigation, the following conditions are assumed for gas mole fractions, area ratio, supply pressure, and supply temperature, respectively: $X_{CO_2} = 0.103$, $X_{N_2} = 0.885$, $X_{H_2O} = 0.012$, $A/A^* = 20.07$, $P_o = 10.2$ atm, and $T_o = 1600°K$. These are typical conditions for CO_2-N_2 gasdynamic lasers. The two-dimensional nozzle shape employs a linear subsonic contour with a 45° entrance angle, α, a prescribed radius of curvature at the nozzle throat, and a 37.5° initial supersonic expansion angle, θ, followed by a minimum length supersonic contour [see Fig. 1b)]. Results are obtained for throat heights, $h^* = 0.3$ mm, 1.0 mm, and 2.0 mm, with radii of curvature equal to 0, 1, 2, and 3 throat heights. Since the nozzles are geometrically similar, a decrease in throat height results in a corresponding decrease in nozzle length. Consequently, larger axial gradients (dp/dx, $d\rho/dx$, etc.) and therefore greater vibrational freezing occurs as the throat heights become smaller.

Typical gain profiles for $h^* = 1.0$ mm, 0.3 mm, and 2.0 mm

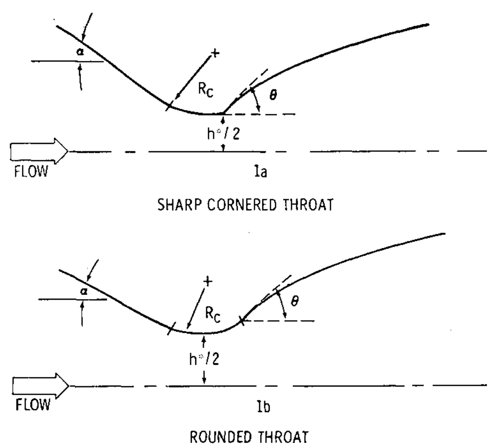

Fig. 1 Typical nozzle contours for gas dynamic lasers.

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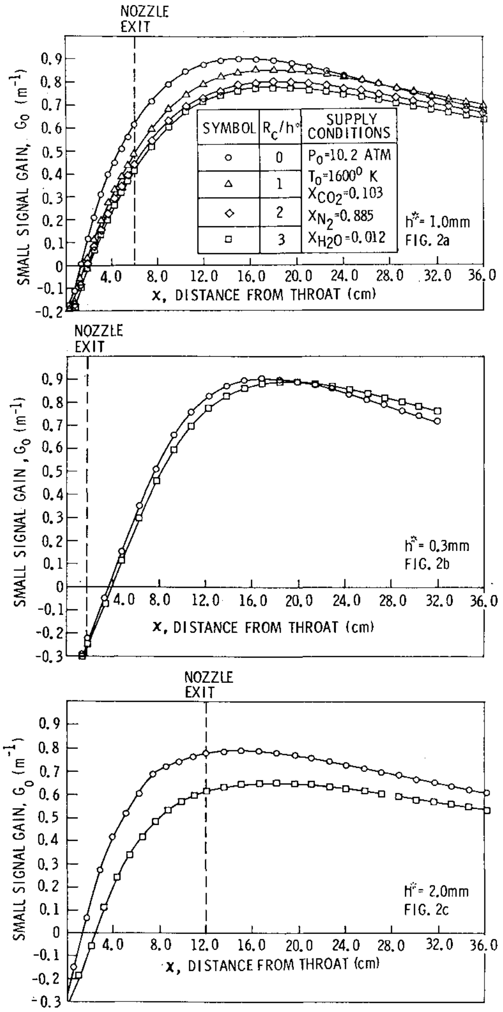

Fig. 2 **Radius of curvature effect on gain along cavity for various throat heights.**

are shown in Fig. 2. As expected, the highest gains occur for R_c/h^* equal to zero (sharp throat) with reduced gains as R_c/h^* increases. Note that for a given throat height the reductions due to rounding the throat are on the order of 0–15%. This relatively small sacrifice in gain as R_c/h^* increases must be weighed against the attendant improvements in ease of manufacture and assembly.

Another physical parameter of interest is the maximum available laser energy per unit mass of gas passing through the nozzle. A detailed analytical derivation of this parameter is given in Ref. 11. A curve of maximum available energy (nondimensionalized by RT_o, where R is the specific gas constant for the mixture and T_o is the reservoir temperature) as a function of throat height and throat radius of curvature is shown in Fig. 3. This curve shows that the losses in available energy for a given throat height with increasing radius of curvature are on the same order as the aforementioned losses in gain.

The current manufacture of accurate nozzle contours with sharp throats is expensive. Our experience has been that the rejection rate of such nozzles is as high as 50%. Also alignment of the individual sharp throat nozzles into a two-dimensional array is critical[6] since a good quality flow downstream of the nozzles is highly desirable. The rounded throats are simpler and cheaper to align, and therefore increase the likelyhood of a good quality flowfield. In this regard, the results shown in Figs. 2 and 3 imply that gasdynamic laser specific power, albeit reduced, is not seriously degraded by complete rounding of the throat (keeping the throat height constant). Of course, the reductions in specific power due

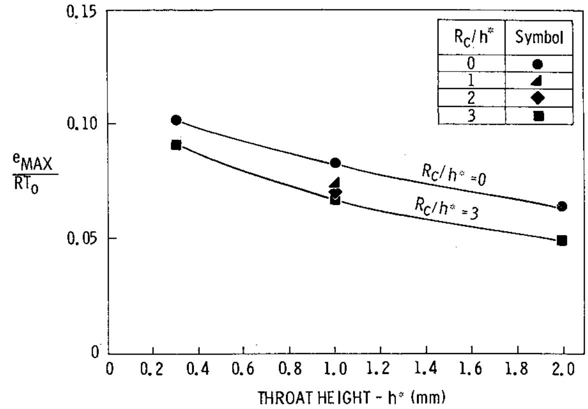

Fig. 3 **Throat height effect on maximum available energy for various radii of curvature.**

to rounding of the throat can be recouped by reducing the throat height; on the other hand, this will most likely negate the practical aerodynamic and manufacturing advantages obtained by rounding of the throat.

It should be emphasized that the present results are obtained from quasi-one-dimensional flow calculations. However, the minimum length nozzles used in practice are two-dimensional, and as such, the fluid elements traveling along different streamlines experience different flow gradients. Thus, the nonequilibrium history of each streamline is different. This prompts a question regarding the limit of applicability of a quasi-one-dimensional analysis for two-dimensional flows. In particular, are the present results for the effect of R_c/h^* meaningful for a flow which is actually two-dimensional? To examine this question, quasi-one-dimensional analytical results for gain are compared with actual two-dimensional measurements in Refs. 9 and 10. Agreement is found to be excellent. As a further check, in Fig. 4, the density profile from the nonequilibrium quasi-one-dimensional analysis (variable γ) is compared to a two-dimensional method of characteristics solution for a fixed $\gamma = 1.40$ and $h^* = 1.0$ mm. This comparison indicates that the quasi-one-dimensional nonequilibrium profile is a reasonable average of two-dimensional results for the nozzle centerline and nozzle wall. In

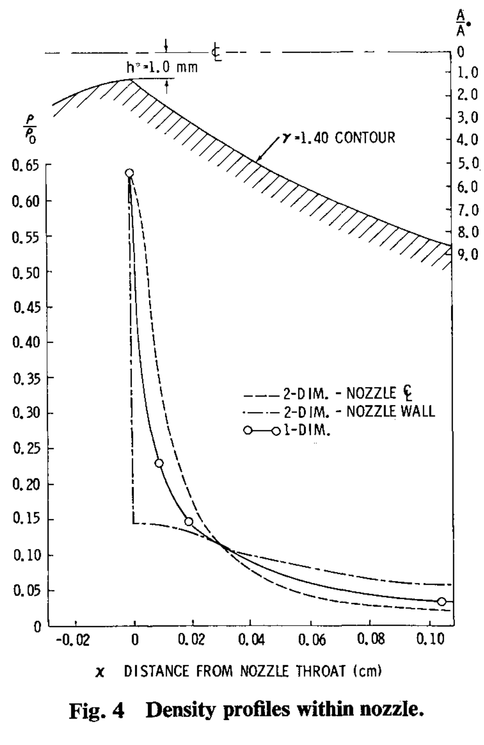

Fig. 4 **Density profiles within nozzle.**

turn, the computed quasi-one-dimensional gain profile should also be a reasonable average of the actual two-dimensional results. Similar trends occur for throat heights = 0.3 mm and 2.0 mm with γ variations between 1.33 and 1.40.[11]

The following conclusion is apparent. In comparison to a completely sharp throat ($R_c = 0$), the present results indicate that gain and maximum available power are reduced on the order of 0–15% (depending on the particular case) by complete rounding of the throat (on both the subsonic and supersonic sides). Whether the manufacturing and alignment advantages of round throats outweighs this relatively small reduction in laser performance depends upon the individual application. However, the purpose of this Note has been to present data which will be helpful in making such decisions.

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Prediction of Tank Pressure History in a Blowdown Propellant Feed System

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Nomenclature

d = propellant tank diameter
g = acceleration of gravity
Gr = Grashof number
H = heat transfer coefficient
K = thermal conductivity
m = mass
Nu = Nusselt number
Pr = Prandtl number
Q = heat content

Subscripts

d = diameter (characteristic length)
p = propellant
w = tank wall

Introduction

MANY current spacecraft utilize the same tankage for propellant and pressurant to avoid the complexity of a separate pressurant tank and, hence, improve reliability; therefore, the initial tank pressure decays as propellant is consumed. Since the flowrate to the rocket engine decreases as the tank pressure decreases, the impulse available at any time prior to propellant depletion is dependent upon the tank pressure history. The observed pressure decay has caused this type of design to be designated the "blowdown type" of propellant supply system, although none of the pressurant is expelled from the tank.

It has been shown[1] that the rate of heat transfer from a sphere to the contained fluid could be accurately predicted by an existing empirical correlation. The same correlation is applied to the blowdown type of propulsion system design to predict transient pressure histories and, hence, the total impulse available at any time.

Discussion

The propellant supply systems shown schematically in Fig. 1 are typical blowdown designs. The propellant leaves the tank at the instantaneous rate of $\dot{m}_p$, and heat is transferred to the pressurant from the pressurant tank wall and propellant surface at the instantaneous rates, $\dot{Q}_w$ and $\dot{Q}_p$, respectively. Since the derivation of the equations describing the controlling thermodynamic processes is similar to that presented in Ref. 1, the derivation will not be given. However, a list of the applicable assumption is in order. They are: 1) the internal energy of the pressurant is a function of temperature only; 2) the pressurant behaves as a perfect gas (a good assumption at the relative low pressures of 20 atmospheres encountered in most designs); 3) specific impulse, flow controlling orifice

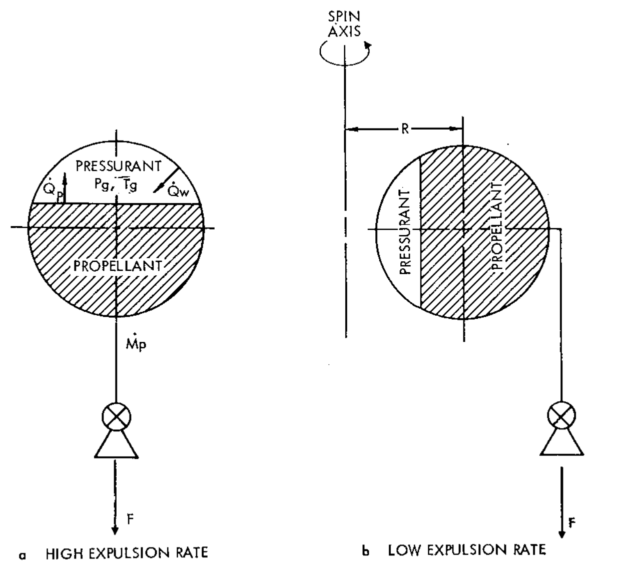

Fig. 1 Blowdown propellant supply system schematics.

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